OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4751**

Introduction to Advanced Mathematics (C1)

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

WARNING

You are not allowed to use a calculator in this paper

This question paper consists of 4 printed pages.

Section A (36 marks)

1 n is a positive integer. Show that $n^2 + n$ is always even. [2]

2

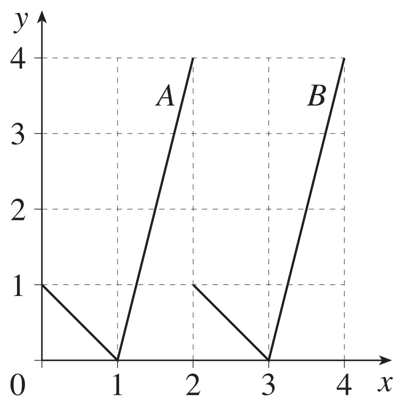


Fig. 2

Fig. 2 shows graphs A and B.

(i) State the transformation which maps graph A onto graph B. [2]

(ii) The equation of graph A is $y = \mathrm{f}(x)$.

Which one of the following is the equation of graph B?

$y = \mathrm{f}(x) + 2$	$y = \mathrm{f}(x) - 2$	$y = \mathrm{f}(x+2)$	$y = \mathrm{f}(x-2)$
$y = 2\mathrm{f}(x)$	$y = \mathrm{f}(x+3)$	$y = \mathrm{f}(x-3)$	$y = 3\mathrm{f}(x)$

[2]

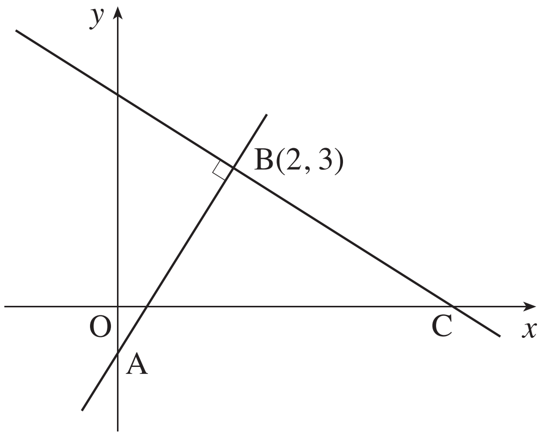
3 Find the binomial expansion of $(2 + x)^4$, writing each term as simply as possible. [4]

4 Solve the inequality $\dfrac{3(2x+1)}{4} > -6$. [4]

5 Make C the subject of the formula $P = \dfrac{C}{C+4}$. [4]

6 When $x^3 + 3x + k$ is divided by $x - 1$, the remainder is 6. Find the value of k. [3]

7



Not to scale

Fig. 7

The line AB has equation $y = 4x - 5$ and passes through the point $B(2, 3)$, as shown in Fig. 7. The line BC is perpendicular to AB and cuts the x-axis at C. Find the equation of the line BC and the x-coordinate of C. [5]

8 **(i)** Simplify $5\sqrt{8} + 4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. [2]

(ii) Express $\dfrac{\sqrt{3}}{6 - \sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are rational. [3]

9 **(i)** Find the range of values of k for which the equation $x^2 + 5x + k = 0$ has one or more real roots. [3]

(ii) Solve the equation $4x^2 + 20x + 25 = 0$. [2]

Section B (36 marks)

10 A circle has equation $x^2 + y^2 = 45$.

(i) State the centre and radius of this circle. [2]

(ii) The circle intersects the line with equation $x + y = 3$ at two points, A and B. Find algebraically the coordinates of A and B.

Show that the distance AB is $\sqrt{162}$. [8]

[Turn over

11 **(i)** Write $x^2 - 7x + 6$ in the form $(x-a)^2 + b$. [3]

(ii) State the coordinates of the minimum point on the graph of $y = x^2 - 7x + 6$. [2]

(iii) Find the coordinates of the points where the graph of $y = x^2 - 7x + 6$ crosses the axes and sketch the graph. [5]

(iv) Show that the graphs of $y = x^2 - 7x + 6$ and $y = x^2 - 3x + 4$ intersect only once. Find the x-coordinate of the point of intersection. [3]

12 **(i)** Sketch the graph of $y = x(x-3)^2$. [3]

(ii) Show that the equation $x(x-3)^2 = 2$ can be expressed as $x^3 - 6x^2 + 9x - 2 = 0$. [2]

(iii) Show that $x = 2$ is one root of this equation and find the other two roots, expressing your answers in surd form.

Show the location of these roots on your sketch graph in part **(i)**. [8]

Mark Scheme 4751
January 2006

Section A

1	$n(n+1)$ seen = odd × even and/or even × odd = even	M1 A1	<u>or</u> B1 for n odd $\Rightarrow n^2$ odd, and comment eg odd + odd = even B1 for n even$\Rightarrow n^2$ even, and comment eg even + even = even allow A1 for 'any number multiplied by the consecutive number is even'	2
2	(i) translation of $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (ii) $y = \mathrm{f}(x-2)$	1 1 2	 or '2 to the right' or '$x \to x + 2$' or 'all x values are increased by 2' 1 for $y = \mathrm{f}(x + 2)$	4
3	$16 + 32x + 24x^2 + 8x^3 + x^4$ isw	4	3 for 4 terms correct, 2 for 3 terms correct, or M1 for 1 4 6 4 1 s.o.i. and M1 for expansion with correct powers of 2	4
4	$x > -4.5$ o.e. isw www [M1 for × 4 M1 expand brackets or divide by 3 M1 subtract constant from LHS M1 divide to find x]	4	accept −27/6 or better; 3 for $x = -4.5$ etc or Ms for each of the four steps carried out correctly with inequality [−1 if working with equation] (ft from earlier errors if of comparable difficulty)	4
5	$[C =]\dfrac{4P}{1-P}$ or $\dfrac{-4P}{P-1}$ o.e.	4	M1 for $PC + 4P = C$ M1 for $4P = C - PC$ or ft M1 for $4P = C(1 - P)$ or ft B3 for $[C =]\dfrac{4}{\frac{1}{P}-1}$ o.e. unsimplified	4
6	f(1) used $1^3 + 3 \times 1 + k = 6$ $k = 2$	M1 A1 A1	or division by $x - 1$ as far as $x^2 + x$ or remainder = $4 + k$ B3 for $k = 2$ www	3
7	grad BC = – ¼ soi $y - 3 = -\frac{1}{4}(x - 2)$ o.e. cao 14 or ft from their BC	2 1 2	M1 for $m_1m_2 = -1$ soi or for grad AB = 4 or grad BC = 1/4 e.g. $y = -0.25x + 3.5$ M1 for subst $y = 0$ in their BC	5
8	(i) $30\sqrt{2}$ (ii) $\dfrac{1}{11}+\dfrac{2}{11}\sqrt{3}$ or $\dfrac{3}{33}+\dfrac{6}{33}\sqrt{3}$ or mixture of these	2 3	M1 for $\sqrt{8}=2\sqrt{2}$ or $\sqrt{50} = 5\sqrt{2}$ soi B1 for $6\sqrt{50}$ or other correct $a\sqrt{b}$ M1 for mult num and denom by $6+\sqrt{3}$ and M1 for denom = 11 or 33	5

			B2 for $\frac{3+6\sqrt{3}}{33}$ or $\frac{1+2\sqrt{3}}{11}$	
9	(i) $k \le 25/4$ (ii) -2.5	3 2	M2 for $5^2 - 4k \ge 0$ or B2 for 25/4 obtained isw or M1 for $b^2 - 4ac$ soi or completing square accept $-20/8$ or better, isw; M1 for attempt to express quadratic as $(2x + a)^2$ or for attempt at quadratic formula	5

Section B

10	**i**	(0, 0), $\sqrt{45}$ isw or $3\sqrt{5}$	1+1		2
	ii	$x = 3 - y$ or $y = 3 - x$ seen or used subst in eqn of circle to eliminate variable $9 - 6y + y^2 + y^2 = 45$ $2y^2 - 6y - 36 = 0$ or $y^2 - 3y - 18 = 0$ $(y - 6)(y + 3) = 0$ $y = 6$ or -3 $x = -3$ or 6 $\sqrt{(6 - -3)^2 + (3 - -6)^2}$	M1 M1 M1 M1 M1 A1 A1 M1	for correct expn of $(3 - y)^2$ seen oe condone one error if quadratic or quad. formula attempted [complete sq attempt earns last 2 Ms] or A1 for (6, −3) and A1 for (−3, 6) no ft from wrong points (A.G.)	8
11	**i**	$(x - 3.5)^2 - 6.25$	3	B1 for $a = 7/2$ o.e, B2 for $b = -25/4$ o.e. or M1 for $6 - (7/2)^2$ or $6 - (\text{their } a)^2$	3
	ii	(3.5, −6.25) o.e. or ft from their (i)	1+1	allow $x = 3.5$ and $y = -6.25$ or ft; allow shown on graph	2
	iii	(0, 6) (1, 0) (6, 0) curve of correct shape fully correct intns and min in 4th quadrant	3 G1 G1	1 each [stated or numbers shown on graph]	5
	iv	$x^2 - 7x + 6 = x^2 - 3x + 4$ $2 = 4x$ $x = ½$ or 0.5 or 2/4 cao	M1 M1 A1	 or $4x - 2 = 0$ (simple linear form; condone one error) condone no comment re only one intn	3
12	**i**	sketch of cubic the correct way up curve passing through (0, 0) curve touching x axis at (3, 0)	G1 G1 G1		3
	ii	$x(x^2 - 6x + 9) = 2$ $x^3 - 6x^2 + 9x = 2$	M1 M1	or $(x^2 - 3x)(x - 3) = 2$ [for one step in expanding brackets] for 2nd step, dep on first M1	2
	iii	subst $x = 2$ in LHS of their eqn or in $x(x - 3)^2 = 2$ o.e. working to show consistent division of their eqn by $(x - 2)$ attempted $x^2 - 4x + 1$	1 1 M1 A1	or 2 for division of their eqn by $(x - 2)$ and showing no remainder or inspection attempted with $(x^2 + kx + c)$ seen	

		soln of their quadratic by formula or completing square attempted $x = 2 \pm \sqrt{3}$ or $(4 \pm \sqrt{12})/2$ isw locating the roots on intersection of their curve and $y = 2$	M1 A2 G1	condone ignoring remainder if they have gone wrong A1 for one correct must be 3 intns; condone x = 2 not marked; mark this when marking sketch graph in (i)	 7 G1

4751: Introduction to Advanced Mathematics (C1) (Written Examination)

General Comments

The candidature was a little smaller than last January, with fewer year 13 students this year compared with those transferring to the new specification then.

There were many excellent scripts, but also a long tail of very weak candidates. It appears that some centres are using this first examination as a 'wake-up call' to weak candidates as to the standards expected during their A level course, as well as hoping that their better candidates gain a good mark on C1 and can concentrate confidently on C2 during the rest of year 12.

A calculator is not allowed in this paper and, as last year, some candidates found this a considerable disadvantage, making errors in basic arithmetic, in particular with negative numbers and fractions. Lack of skill in factorising was apparent, with many candidates resorting to the quadratic formula in questions 9 and 10, for instance.

Using graph paper to draw graphs when a sketch graph has been requested remains an issue. Some candidates not only waste time in drawing such graphs, but also tend to use too large a vertical scale and too few points to achieve a good shape. It is acknowledged that the present listing of graph paper as additional materials on the front of the paper has not helped this situation.

In general, time was not an issue. A few candidates petered out towards the end of question 12, but there were very few for whom this appeared to be because they had run out of time.

Comments on Individual Questions

Section A

1) This proved to be a testing starter. Most candidates used a few different values, often some odd and some even – several commented that they had thereby proved the result by exhaustion! In the better solutions there was a realisation that generalisation was required, with good arguments being produced, the $n^2 + n$ being more popular than the $n(n + 1)$ form. A few good candidates substituted $2k$ and $2k + 1$.

2) In part (i), many did not realise that the word 'translation' was needed to describe the transformation and simply wrote 'moves by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$' or equivalent. In part (ii) most had the correct answer, but the distractors were also used.

3) Many candidates used the binomial theorem successfully, with usually just the weaker candidates not knowing how to proceed. Very few tried to multiply out the brackets, but those who did were often successful.

4) The majority of candidates scored at least 3 of the 4 marks for solving this inequality. The steps which caused the most difficulty were multiplying by 4, and subtracting 3 from −24. The mark scheme allowed for follow-through from wrong steps.

5) Better students had no problems in changing the subject of this formula, but many candidates had no idea how to proceed after $PC + 4P = C$, whilst many did not get this far and appeared to have no strategy at all.

6) Use of the remainder theorem nearly always led to full marks. In some centres, long division was popular, with many candidates unable to cope with the first subtraction required and gaining no marks, although there were also quite a few candidates who used this method successfully.

7) There was a lot of difference between centres in candidates' responses here. A good number of candidates gained full marks, but there were also many arithmetical errors, either in simplifying a correct equation, or in proceeding from $\frac{1}{4}x = 3\frac{1}{2}$ via $x = \frac{3\frac{1}{2}}{\frac{1}{4}}$. Many missed out the last part or used $x = 0$ instead of $y = 0$. Some of the weaker candidates had no idea about the condition $m_1m_2 = -1$ for perpendicular lines.

8) Better candidates had no problem with the first part, but errors in manipulating the surds here were frequent, with common errors being work such as $4\sqrt{50} = 4 + 5\sqrt{2} = 9\sqrt{2}$ or $5\sqrt{8} = 5\sqrt{4 \times 2} = 20\sqrt{2}$. Weaker candidates often had little idea how to proceed here. Most knew the method for rationalising the denominator in the second part, but many stopped at $\frac{3 + 6\sqrt{3}}{33}$ or made errors after this in obtaining the required form.

9) This was only completed successfully by the best candidates, with quite a few of the better ones getting as far as $k < 25/4$ but omitting the equality. Weaker candidates perhaps knew the answer had something to do with $b^2 - 4ac$ and gained a method mark or simply tried a number of values of k or omitted the question. In part (ii), disappointingly few immediately recognised the perfect square. Those who attempted the quadratic formula often made errors in working out $4 \times 4 \times 25$ or were bemused when the discriminant was zero, though many used it correctly.

Section B

10 (i) This was mostly well answered. A few became muddled and thought the centre was (3, 6) or (6, 3); a small number stated the radius as 45.

(ii) A majority of the candidates understood how to eliminate a variable from the equation of the circle in order to produce a quadratic equation in one variable. This was usually done accurately and led to the correct answer. A few errors did occur by making the wrong substitution (e.g. y = x – 3), or by expanding $(3 - x)^2$ incorrectly. Once they had reached the correct quadratic equation, the correct solution generally followed. A larger number of candidates used the formula to solve the resulting quadratic, rather than factorising – with mixed results, depending on how well they remembered the formula - many incorrect versions were quoted and used. Candidates often worked with the harder equation $2x^2$ –

$6x - 36 = 0$ (or its equivalent in y) instead of first dividing through by 2, which would have simplified both factorising and use of the formula.

A few candidates combined the two equations without eliminating a variable e.g. $x^2 + y^2 - 45 = x + y - 3$ and were therefore unable to proceed any further.

Those who achieved the correct solution rarely failed to calculate the distance AB correctly.

11 (i) Although the best candidates did this correctly there was evidence to suggest that many of them are unable to calculate $(^7/_2)^2$. So although they understood what to do they were let down by their poor arithmetical ability. Some candidates added an =' to the expression, taking the 6 'across ', subtracting it from 3.5^2 and then 'taking it back'. Those who used this approach often forgot to negate their result and so lost out – the examiners do not recommend this method.

(ii) Many candidates realised that there was a connection between this part and the first and generally their answers did follow through, though there were occasional errors in sign. Other candidates started all over again, sometimes using calculus. Many of these attempts were unsuccessful, particularly in calculating the *y*-coordinate.

(iii) Most managed to find at least one of the intercepts, with a good number finding all three. The graph sketching was not done particularly well. The curves were often not very smooth, nor were they symmetrical.. Many candidates plotted the 'intercept' points first and tried to fit a curve to them, which is not an easy task. This is one reason why the use of graph paper may not have been their best option.

(iv) Most candidates were able to make a good attempt at this part and were usually successful, even if they had failed to pick up many marks in the earlier parts. However, after a correct first step, a significant minority made errors in simplifying and solving their equation.

12 (i) The sketches were again disappointing but some examiners reported that they were more successful that in the previous question. The shape was more often correct this time. but often not both through (0, 0) and tangential to (3, 0). A few candidates drew parabolas or negative cubic shapes. Weaker candidates did not see the usefulness of the given form of the equation, expanding and working out coordinates rather than seeing the roots by inspection.

(ii) Most candidates were able to gain the marks here, with some having already done part of the work for this in part (i) and just a few making sign or other errors as they attempted to reach the given answer.

(iii) Most candidates used the factor theorem to show that x = 2 was a root of the equation – a small number of candidates incorrectly tried f(−2). Other candidates relied on the long division process which they used in the next part of the question. The division was generally done correctly – the source of any errors generally came from errors in dealing with negative numbers. Those who used inspection were usually successful. Having found the quadratic factor most candidates made a reasonable attempt at deriving the other two solutions. Full credit was given for getting as far as $\frac{4 \pm \sqrt{12}}{2}$; candidates benefited from this since various arithmetic slips occurred in their attempts to simplify it.

The final mark for showing the location of these roots on their sketch was very rarely earned. The *x*-values were sometimes marked on the *x*-axis rather than on the curve; occasionally a further cubic was drawn, translated by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (which was not what candidates were asked to do).